APPENDIX A

DESIGN METHODS AND EQUATIONS

A. Introduction

This appendix contains explanations of the equations and methods used to develop the design charts of this publication, where those equations and methods are not fully described in the main text. The following topics are discussed: the design equations for the unsubmerged and submerged inlet control nomographs, the dimensionless design curves for culvert shapes and sizes without nomographs, and the dimensionless critical depth charts for long span culverts and corrugated metal box culverts.

B. Inlet Control Nomograph Equations

The design equations used to develop the inlet control nomographs are based on the research conducted by the National Bureau of Standards (NBS) under the sponsorship of the Bureau of Public Roads (now the Federal Highway Administration). Seven progress reports were produced as a result of this research. Of these, the first and fourth through seventh reports dealt with the hydraulics of pipe and box culvert entrances, with and without tapered inlets (4, 7, to 10). These reports were one source of the equation coefficients and exponents, along with other references and unpublished FHWA notes on the development of the nomographs (56 and 57).

The two basic conditions of inlet control depend upon whether the inlet end of the culvert is or is not submerged by the upstream headwater. If the inlet is not submerged, the inlet performs as a weir. If the inlet is submerged, the inlet performs as an orifice. Equations are available for each of the above conditions.

Between the unsubmerged and the submerged conditions, there is a transition zone for which the NBS research provided only limited information. The transition zone is defined empirically by drawing a curve between and tangent to the curves defined by the unsubmerged and submerged equations. In most cases, the transition zone is short and the curve is easily constructed.

Table 8 contains the unsubmerged and submerged inlet control design equations. Note that there are two forms of the unsubmerged equation. Form (1) is based on the specific head at critical depth, adjusted with two correction factors. Form (2) is an exponential equation similar to a weir equation. Form (1) is preferable from a theoretical standpoint, but Form (2) is easier to apply and is the only documented form of equation for some of the inlet control nomographs.

The constants and the corresponding equation form are given in Table 9. Table 9 is arranged in the same order as the design nomographs in Appendix D, and provides the unsubmerged and submerged equation coefficients for each shape, material, and edge configuration. For the unsubmerged equations, the form of the equation is also noted.

Table 8--Inlet Control Design Equations.

UNSUBMERGED[1]

$$\text{Form}(1)\ \frac{HW_i}{D} = \frac{H_c}{D} + K\left[\frac{K_u Q}{AD^{0.5}}\right]^M - 0.5S^2 \qquad (26)$$

$$\text{Form}(2)\ \frac{HW_i}{D} = K\left[\frac{K_u Q}{AD^{0.5}}\right]^M \qquad (27)$$

SUBMERGED[3]

$$\frac{HW_I}{D} = c\left[\frac{K_u Q}{AD^{0.5}}\right]^2 + Y - 0.5S^2 \qquad (28)$$

Definitions

HW_i	Headwater depth above inlet control section invert, m (ft)
D	Interior height of culvert barrel, m (ft)
H_c	Specific head at critical depth ($d_c + V_c^2/2g$), m (ft)
Q	Discharge, m^3/s (ft^3/s)
A	Full cross sectional area of culvert barrel, m^2 (ft^2)
S	Culvert barrel slope, m/m (ft/ft)
K,,M,c,Y	Constants from Table 9
K_u	1.811 SI (1.0 English)

NOTES: [1]Equations (26) and (27) (unsubmerged) apply up to about $Q/AD^{0.5}$ = 1.93 (3.5 English)
[2]For mitered inlets use +0.7S instead of -0.5S as the slope correction factor
[3]Equation (28) (submerged) applies above about $Q/AD^{0.5}$ = 2.21 (4.0. English)

The equations may be used to develop design curves for any conduit shape or size. Careful examination of the equation constants for a given form of equation reveals that there is very little difference between the constants for a given inlet configuration. Therefore, given the necessary conduit geometry for a new shape from the manufacturer, a similar shape is chosen from Table 9, and the constants are used to develop new design curves. The curves may be quasi-dimensionless, in terms of $Q/AD^{0.5}$ and HW_i /D, or dimensional, in terms of Q and HW_i for a particular conduit size. To make the curves truly dimensionless, $Q/AD^{0.5}$ must be divided by $g^{0.5}$, but this results in small decimal numbers. Note that coefficients for rectangular (box) shapes should not be used for nonrectangular (circular, arch, pipe-arch, etc.) shapes and vice-versa. A constant slope value of 2 percent (0.02) is usually selected for the development of design curves. This is because the slope effect is small and the resultant headwater is conservatively high for sites with slopes exceeding 2 percent (except for mitered inlets).

Table 9--Constants for Inlet Control Design Equations.

Chart No.	Shape and Material	Nomograph Scale	Inlet Edge Description	Equation Form	Unsubmerged K	Unsubmerged M	Submerged c	Submerged Y	References
1	Circular Concrete	1	Square edge w/headwall	1	.0098	2.0	.0398	.67	56/57
		2	Groove end w/headwall		.0018	2.0	.0292	.74	
		3	Groove end projecting		.0045	2.0	.0317	.69	
2	Circular CMP	1	Headwall	1	.0078	2.0	.0379	.69	56/57)
		2	Mitered to slope		.0210	1.33	.0463	.75	
		3	Projecting		.0340	1.50	.0553	.54	
3	Circular	A	Beveled ring, 45° bevels	1	.0018	2.50	.0300	.74	57
		B	Beveled ring, 33.7° bevels*		.0018	2.50	.0243	.83	
8	Rectangular Box	1	30° to 75° wingwall flares	1	.026	1.0	.0347	.81	56
		2	90° and 15° wingwall flares		.061	.75	.0400	.80	56
		3	0° wingwall flares		.061	.75	.0423	.82	8
9	Rectangular Box	1	45° wingwall flare d = .043D	2	.510	.667	.0309	.80	8
		2	18° to 33.7° wingwall flare d = .083D		.486	.667	.0249	.83	
10	Rectangular Box	1	90° headwall w/3/4" chamfers	2	.515	.667	.0375	.79	8
		2	90° headwall w/45° bevels		.495	.667	.0314	.82	
		3	90° headwall w/33.7° bevels		.486	.667	.0252	.865	
11	Rectangular Box	1	3/4" chamfers; 45° skewed headwall	2	.545	.667	.04505	.73	8
		2	3/4" chamfers; 30° skewed headwall		.533	.667	.0425	.705	
		3	3/4" chamfers; 15° skewed headwall		.522	.667	.0402	.68	
		4	45° bevels; 10°-45° skewed headwall		.498	.667	.0327	.75	
12	Rectangular Box 3/4" chamfers	1	45° non-offset wingwall flares	2	.497	.667	.0339	.803	8
		2	18.4° non-offset wingwall flares		.493	.667	.0361	.806	
		3	18.4° non-offset wingwall flares 30° skewed barrel		.495	.667	.0386	.71	
13	Rectangular Box Top Bevels	1	45° wingwall flares - offset	2	.497	.667	.0302	.835	8
		2	33.7° wingwall flares - offset		.495	.667	.0252	.881	
		3	18.4° wingwall flares - offset		.493	.667	.0227	.887	
16-19	C M Boxes	2	90° headwall	1	.0083	2.0	.0379	.69	57
		3	Thick wall projecting		.0145	1.75	.0419	.64	
		5	Thin wall projecting		.0340	1.5	.0496	.57	

Table 9 (continued) Constants for Inlet Control Design Equations

Chart No.	Shape and Material	Nomograph Scale	Inlet Edge Description	Equation Form	Unsubmerged		Submerged		References
					K	M	c	Y	
29	Horizontal	1	Square edge w/headwall	1	.0100	2.0	.0398	.67	57
	Ellipse	2	Groove end w/headwall		.0018	2.5	.0292	.74	
	Concrete	3	Groove end projecting		.0045	2.0	.0317	.69	
30	Vertical	1	Square edge w/headwall	1	.0100	2.0	.0398	.67	57
	Ellipse	2	Groove end w/headwall		.0018	2.5	.0292	.74	
	Concrete	3	Groove end projecting		.0095	2.0	.0317	.69	
34	Pipe Arch	1	90° headwall	1	.0083	2.0	.0379	.69	57
	18" Corner	2	Mitered to slope		.0300	1.0	.0463	.75	
	Radius CM	3	Projecting		.0340	1.5	.0496	.57	
35	Pipe Arch	1	Projecting	1	.0300	1.5	.0496	.57	56
	18" Corner	2	No Bevels		.0088	2.0	.0368	.68	
	Radius CM	3	33.7° Bevels		.0030	2.0	.0269	.77	
36	Pipe Arch	1	Projecting	1	.0300	1.5	.0496	.57	56
	31" Corner		No Bevels		.0088	2.0	.0368	.68	
	Radius CM		33.7° Bevels		.0030	2.0	.0269	.77	
41-43	Arch CM	1	90° headwall	1	.0083	2.0	.0379	.69	57
		2	Mitered to slope		.0300	1.0	.0463	.75	
		3	Thin wall projecting		.0340	1.5	.0496	.57	
55	Circular	1	Smooth tapered inlet throat	2	.534	.555	.0196	.90	3
		2	Rough tapered inlet throat		.519	.64	.0210	.90	
56	Elliptical	1	Tapered inlet-beveled edges	2	.536	.622	.0368	.83	3
	Inlet Face	2	Tapered inlet-square edges		.5035	.719	.0478	.80	
		3	Tapered inlet-thin edge projecting		.547	.80	.0598	.75	
57	Rectangular	1	Tapered inlet throat	2	.475	.667	.0179	.97	3
58	Rectangular	1	Side tapered-less favorable edges	2	.56	.667	.0446	.85	3
	Concrete	2	Side tapered-more favorable edges		.56	.667	.0378	.87	
59	Rectangular	1	Slope tapered-less favorable edges	2	.50	.667	.0446	.65	3
	Concrete		Slope tapered-more favorable edges		.50	.667	.0378	.71	